

# The Universe as a Diffusive Medium – Constraining or Detecting the Gravitational Wave Background

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We calculate the “seeing” effect on distant sources due to a gravitational wave background. We derive the limit in strain and energy density of the gravitational wave based on the limit of detectability of this effect with the present day telescope resolution. We also compare our detection limit to those obtained from existing methods.

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## I. INTRODUCTION

The generation of gravitational waves is believed to be an ongoing process in the evolution of the universe. Presently, aspherical supernovae and the merger of compact binaries are probably the two most common (or at least prosaic) important sources of gravitational waves. Gravity waves may also have been copiously produced in the early universe and would probably have led to a nearly homogeneous and isotropic background of gravity waves. The most well-motivated model predicting a gravitational wave background from the early universe is inflation (Grishchuk; Ford and Parker; Starobinsky; Rubakov, Sahin and Veryaskin; Fabbri and Pollock; Abbot and Wise; Starobinsky; Abbot and Shaefer; Abbot and Harari; Allen; Ressel and Turner; Sahni; Souradeep and Sahni; Liddle and Lyth; Davis et al.; Salopek; Lucchin, Matarrese and Mollerach; Dolgov and Silk; Turner; Crittenden et. al.; Harari and Zaldarriaga; Crittenden, Davis and Steinhardt; Ng and Ng; Krauss & White; White; White, Krauss and Silk; Bond et. al.; Grischuk; Falk, Rangarajan and Srednicki; Luo and Schramm; Srednicki).

The effect of this gravitational wave background on pulsar timing measurements has already been investigated (Bertotti *et al.* 1983). It has been also been studied in the context of the cosmic microwave background radiation when computing the Sachs-Wolfe contribution (Krauss & White 1992; Davis *et al.* 1992) and the polarization of the radiation (Polnarev 1985; Crittenden *et al.* 1993). Fakir (1993) has shown how individual gravity waves bend lightlike geodesics. For distant objects such as quasars, one therefore expects the gravitational wave background to perturb the light coming to us and distort the image, creating a “seeing” effect. The detection (or non-detection) of this effect provides us with an important additional constraint on models predicting the existence of a gravitational wave background.

We present in the following sections a calculation of the expected deviation of light rays due to a gravitational wave background. We begin by recalculating the null geodesic deviation due to a single gravitational wave. We proceed to derive the expected RMS deviation, with the deviation modeled as a random walk in three-dimensional space. We use that to put an upper limit on the dimensionless strain,  $h$ , and on the ratio,  $\Omega_g$ , of energy density in the gravitational wave background to the critical energy density required to close the universe. This calculation is valid only if the dimension of the source is larger than the wavelength of the gravitational wave.

## II. DEVIATION OF LIGHT BY A GRAVITATIONAL WAVE

For a gravitational wave propagating in a Minkowski background spacetime, the metric can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

where  $\eta_{\mu\nu}$  is the Minkowski metric and  $h_{\mu\nu}$  is the perturbation. The plane wave solution to Einstein's equations can then be written as:

$$h_{\mu\nu}(x) = h \left( e_{\mu\nu} \exp[ik_\lambda x^\lambda] + e_{\mu\nu}^* \exp[-ik_\lambda x^\lambda] \right) \quad (2)$$

with  $k$  null,

$$k_\mu k^\mu = 0, \quad (3)$$

and

$$k_\mu e_\nu^\mu = \frac{1}{2} k_\nu e_\mu^\mu \quad (4)$$

in harmonic gauge. The polarization tensor  $e_{\mu\nu}$  is symmetric, i.e.  $e_{\mu\nu} = e_{\nu\mu}$ .

For gravitational radiation travelling in the  $-x$  direction

$$k^\mu = k(-1, -1, 0, 0). \quad (5)$$

The four-velocity of a null ray leaving the source in a general direction is given by:

$$u^\mu = (-1, \cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi). \quad (6)$$

The deviation of the null ray leaving the source is computed using the equations of parallel transport for the photon velocity  $u^\mu \equiv dx^\mu/d\lambda$ , where  $\lambda$  is the affine parameter along the null geodesic:

$$\frac{du^\mu}{d\lambda} = -\Gamma_{\alpha\beta}^\mu u^\alpha u^\beta, \quad (7)$$

To leading order in  $h$ ,

$$\Gamma_{\alpha\beta}^\mu = h \frac{i}{2} \eta^{\mu\nu} \{ \exp[ik_\lambda x^\lambda] (k_\beta e_{\nu\alpha} + k_\alpha e_{\nu\beta} - k_\nu e_{\alpha\beta}) - \exp[-ik_\lambda x^\lambda] (k_\beta e_{\nu\alpha}^* + k_\alpha e_{\nu\beta}^* - k_\nu e_{\alpha\beta}^*) \}. \quad (8)$$

Since the gravitational wave is propagating in the  $-x$  direction, equation (4) implies

$$e_{ty} = -e_{xy}, \quad e_{tz} = -e_{xz}, \quad e_{tx} = -(e_{xx} + e_{tt})/2, \quad e_{yy} = -e_{zz}. \quad (9)$$

Taking the polarization to have only y and z components, the only non-vanishing Christoffels are (up to  $\Gamma_{\alpha\beta}^\mu = \Gamma_{\beta\alpha}^\mu$ )

$$\begin{aligned} \Gamma_{yy}^t &= -\Gamma_{zz}^t = \Gamma_{ty}^y = -\Gamma_{tz}^z = \frac{1}{2}h_{yy,t}, \\ -\Gamma_{yy}^x &= \Gamma_{zz}^x = -\Gamma_{zx}^z = \Gamma_{yx}^y = \frac{1}{2}h_{yy,x}, \\ \Gamma_{yz}^t &= \Gamma_{tz}^y = \Gamma_{ty}^z = \frac{1}{2}h_{yz,t}, \\ -\Gamma_{yz}^x &= \Gamma_{xz}^y = \Gamma_{xy}^z = \frac{1}{2}h_{yz,x}, \end{aligned} \quad (10)$$

Therefore, the deviation of the null velocity vector is seen to be of  $\mathcal{O}(h)$ , and is given by:

$$\begin{aligned} \frac{du^t}{d\lambda} &= \frac{1}{2}h_{yy,t}(u^{z2} - u^{y2}), \\ \frac{du^x}{d\lambda} &= \frac{1}{2}h_{yy,x}(-u^{z2} + u^{y2}), \\ \frac{du^y}{d\lambda} &= -(h_{yy,x}u^yu^x + h_{yy,t}u^tu^y + h_{yz,t}u^tu^z + h_{yz,x}u^xu^z), \\ \frac{du^z}{d\lambda} &= (h_{yy,x}u^zu^x + h_{yy,t}u^tu^z - h_{yz,t}u^tu^y - h_{yz,x}u^xu^y). \end{aligned} \quad (11)$$

For null rays parallel or antiparallel to the gravitational wave, this implies that the ray will not be subject to any deviation, as expected. Maximum deviation occurs when the incidence angle between the null ray and the gravitational wave is of  $\pi/2$ . The null vector in this case is  $(-1, 0, 0, 1)$  and:

$$\begin{aligned} \frac{du^t}{d\lambda} &= \frac{1}{2}h_{yy,t}, \\ \frac{du^x}{d\lambda} &= -\frac{1}{2}h_{yy,x}, \\ \frac{du^y}{d\lambda} &= h_{yz,t}, \\ \frac{du^z}{d\lambda} &= -h_{yz,t}. \end{aligned} \quad (12)$$

As the photon travels through a wave train, its direction oscillates with angular amplitude  $\alpha \simeq h$ .

### III. TOTAL DEVIATION OF LIGHT RAY BY RANDOM WALK PROCESS

The deviation of the null geodesic due to a single gravitational wave can be viewed as a single step in a random walk in three dimension. It is therefore possible to evaluate the magnitude of the expected deviation of the light of a distant source caused by a background of gravitational waves. The random walk gives us the following relation:

$$\langle \Delta\theta_{tot} \rangle = \sqrt{\frac{N}{3}}\bar{\alpha}, \quad (13)$$

where  $\bar{\alpha} \sim h$  from the calculation above.  $N$  is the number of average-size gravity wavetrains through which the light has passed and equals the ratio of the source-observer distance,  $d_s$ , to the average coherence length of the gravity wave background,  $\ell_{coh}$ . i.e.  $N = d_s/\ell_{coh}$ . The total deviation can be written as:

$$\Delta\theta_{tot} = 10^{-12} \frac{1}{\sqrt{\omega\ell_{coh}}} \sqrt{\frac{\omega d_s}{300 \text{ Mpc Hz}}} \left(\frac{h}{10^{-20}}\right), \quad (14)$$

where  $\omega$  for primordial gravitational waves ranges from the scale of the present horizon ( $\sim 6000$  Mpc) down to microphysical scales. In this case, we have assumed that the average perturbation was of order  $h$ , which is independent of frequency. Returning to the original perturbation equations, one sees that in fact they are frequency dependent since  $h_{,x}$  and  $h_{,t}$  depend on the wave number, i.e.  $h = h(w)$ .

The ratio of the energy density in the gravitational wave background at a given frequency,  $\omega$ , to the critical density for closure  $\rho_c = 1.054 \times 10^{-5} h_0^2 \text{ GeV/cm}^3$ , is

$$\Omega_g(\omega) \equiv \frac{1}{\rho_c} \frac{d\rho}{d\log \omega} = 3.17 \times 10^{-6} h_0^{-2} \left( \frac{\omega}{Hz} \right)^2 \left( \frac{h}{10^{-20}} \right)^2. \quad (15)$$

This can be rewritten as

$$h_{20}^2(\omega) \equiv \left( \frac{h}{10^{-20}} \right)^2 = 12 \Omega_g(\omega) \left( \frac{H_0}{2\omega} \right)^2, \quad (16)$$

where  $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

#### IV. DETECTABILITY

A photon emitted by a distant object and propagating along the line of sight through the gravitational wave background, will be displaced in the sky compared to its position in the absence of the background, by an angle of the order of  $\Delta\theta_{tot}$ . This displacement should vary as the gravitational wave travels across space, creating a seeing disk for images of distant objects, such as distant galaxies or quasars. This angular deviation should be observable with high enough resolution. With the present high resolution telescopes such as the Hubble Space Telescope, it is easy to put an upper limit on the strength of primordial gravitational waves if the seeing effect is not observed. The Hubble Space Telescope has an angular resolution of the order of  $\Delta\theta_{lim} \sim 1 \text{ arcsec} \sim 5 \times 10^{-7}$  radians. This limit can be pushed down even more in the future with long baseline interferometers ( $\Delta\theta_{lim} \sim 10^{-3} \text{ arcsec} \sim 5 \times 10^{-9}$  radians) or even space-based interferometers ( $\Delta\theta_{lim} \sim 10^{-7} \text{ arcsec} \sim 5 \times 10^{-13}$  radians) built to observe radio sources.

Given that the seeing effect described above hasn't been seen with the present detection limit of  $\Delta\theta_{lim} \simeq 5 \times 10^{-7}$ , we can infer an upper limit on the dimensionless strain or energy density of the background gravitational wave from the condition  $\Delta\theta_{tot} < \Delta\theta_{lim}$  which translates into:

$$h < 2.5 \times 10^{-8} \Delta\theta_{lim} \sqrt{\ell_{coh}/\lambda} \sqrt{\frac{300 \text{ Mpc Hz}}{\omega d_s}}, \quad (17)$$

and

$$\Omega_g h_0^2 < 1.98 \times 10^2 \left( \frac{\omega}{Hz} \right) (\ell_{coh}/\lambda) \left( \frac{3 \text{ Gpc}}{d_s} \right) \left( \frac{\Delta\theta_{lim}}{10^{-8}} \right)^2 \quad (18)$$

It is interesting to note that the dimensionless strain predicted by inflation (Bar-Kana 1994) ranges from  $10^{-16}$  for a frequency,  $\nu$ , of  $10^{-8} \text{ Hz}$  to  $10^{-28}$  for a frequency of  $10^4 \text{ Hz}$ , assuming a strictly scale-invariant spectrum (no tensor

fluctuations). The best limits currently quoted are from pulsar timing, which probes strains greater than  $10^{-13}$  over a small range in frequency near  $\sim 10^{-8}$  Hz. The limit on the energy density of a gravitational wave background in this frequency range is therefore  $\Omega_g h_0^2 < 9 \times 10^{-8}$ .

The measurement of the quadrupole anisotropy produced by a gravitational wave of the cosmic microwave background has also been proposed as a way of probing the strain of the background of gravitational wave although it is difficult to separate the observed signal caused by the density perturbations and the one due to the gravitational wave.

Our method is an alternative astrophysical method and has the advantage of probing smaller strains and a wide range of frequencies. Taking as our representative object a quasar at 3 Gpc  $h_0^{-1}$ , and using  $\Delta\theta_{tot} \leq 5 \times 10^{-7}$ , the new limits on  $h$  range approximately from  $3 \times 10^{-11}$  for a frequency,  $\nu$ , of  $10^{-8}$  Hz to  $3 \times 10^{-17}$  for a frequency of  $10^4$  Hz. As seen in Fig. 1, it gives weaker limits than the pulsar timing method for the same small range of frequency but covers a much wider range of frequencies. As minimum angular resolutions improve, with the development of long baseline inteferometers, these limits could improve, by as much as  $10^3$ . Ultimately, it might be possible to use this method to detect the background of gravitational waves.

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FIG. 1. Limits on the dimensionless strain. The solid line is from gravitational seeing for an object  $3 \text{ Gpc } h_0^{-1}$  distant and assuming  $\ell_{coh}/\lambda = 1$ . The dotted line is the limit from pulsar timing. The dashed line is a typical inflationary prediction (Bar-Kana 1994).

